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# PROBABILITY, RANDOM VARIABLES, AND STOCHASTIC PROCESSES

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## 232 GENERAL CONCEPTS

## Memoryless Systems

A system is called memoryless if its output is given by

$$y(t) = g[x(t)]$$

where  $g(x)$  is a function of  $x$ . Thus, at a given time  $t = t_1$ , the output  $y(t_1)$  depends only on  $x(t_1)$  and not on any other past or future values of  $x(t)$ .

From the above it follows that the first-order density  $f_y(y; t)$  of  $y(t)$  can be expressed in terms of the corresponding density  $f_x(x; t)$  of  $x(t)$  as in Sec. 5-2. We note, in particular, that

$$E\{y(t)\} = \int_{-\infty}^{\infty} g(x) f_x(x; t) dx$$

Similarly, since  $y(t_1) = g[x(t_1)]$  and  $y(t_2) = g[x(t_2)]$ , the second-order density  $f_y(y_1, y_2; t_1, t_2)$  of  $y(t)$  can be determined in terms of the corresponding density  $f_x(x_1, x_2; t_1, t_2)$  of  $x(t)$  as in Sec. 6-3. In particular

$$E\{y(t_1)y(t_2)\} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x_1 x_2 g(x_1) g(x_2) f_x(x_1, x_2; t_1, t_2) dx_1 dx_2$$

The  $n$ th order density  $f_y(y_1, \dots, y_n; t_1, \dots, t_n)$  of  $y(t)$  can be determined from the corresponding density of  $x(t)$  as in (8-8) where the underlying transformation is the system

$$y(t_i) = g[x(t_i)], \dots, y(t_n) = g[x(t_n)] \quad (9-86)$$

**Stationarity** Suppose that the input to a memoryless system is an SSS process  $x(t)$ . We shall show that the resulting output  $y(t)$  is also SSS.

**PROOF** To determine the  $n$ th order density of  $y(t)$ , we solve the system

$$g(x_i) = y_i, \dots, g(x_n) = y_n \quad (9-87)$$

If this system has a unique solution, then [see (8-8)]

$$f_y(y_1, \dots, y_n; t_1, \dots, t_n) = \frac{f_x(x_1, \dots, x_n; t_1, \dots, t_n)}{|g'(x_1) \cdots g'(x_n)|} \quad (9-88)$$

From the stationarity of  $x(t)$  it follows that the numerator in (9-88) is invariant to a shift of the time origin. And since the denominator does not depend on  $t$ , we conclude that the left side does not change if  $t_i$  is replaced by  $t_i + c$ . Hence,  $y(t)$  is SSS. We can similarly show that this is true even if (9-87) has more than one solution.

**Notes** 1. If  $x(t)$  is stationary of order  $N$ , then  $y(t)$  is stationary of order  $N$ .

2. If  $x(t)$  is stationary in an interval, then  $y(t)$  is stationary in the same interval.

3. If  $x(t)$  is WSS stationary, then  $y(t)$  might not be stationary in any sense.

Square  
output eq

We sh  
If  $y >$   
 $y(x) = \pm 2$

If  $y_1 > 0$  and

has the f  
 $\pm 4\sqrt{y_1 y_2}$

where the  
We na  
 $f_x(x_1, x_2; t)$   
dependent

Example  
autocor  
If  $y$

We

PROOF  
(7-37)]

and (9-8)

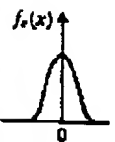


Figure 9-9

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